Resolução comentada Lista 01

01. $\begin{vmatrix} x & x+1 \\ y & y+1 \end{vmatrix} = x(y+1) - y(x+1) = xy + x - xy - y = x - y$.

02.

$$\begin{vmatrix} 2x & x-2 \\ 4x+5 & 3x-1 \end{vmatrix} = 11$$

$$2x \cdot (3x-1) - (x-2) \cdot (4x+5) = 11$$

$$6x^2 - 2x - (4x^2 + 5x - 8x - 10) = 11$$

$$6x^2 - 2x - 4x^2 - 5x + 8x + 10 = 11$$

$$2x^2 + x - 1 = 0 \qquad \text{(Bháskara)}$$

$$x = -1 \text{ ou } x = \frac{1}{2}$$

03.

$$\begin{vmatrix} x-1 & 2 & x \\ 0 & 1 & -1 \\ 3x & x+1 & 2x \end{vmatrix} = \begin{vmatrix} 3x & 2x \\ 4 & -x \end{vmatrix}$$

$$\begin{vmatrix} x-1 & 2 & x \\ 0 & 1 & -1 \\ 3x & x+1 & 2x \end{vmatrix} \begin{matrix} x-1 & 2 \\ 0 & 1 \\ 3x & x+1 \end{matrix} = -3x^2 - 8x$$

$$2x^2 - 2x - 6x - 3x^2 + x^2 - 1 = -3x^2 - 8x$$

$$3x^2 = 1$$

$$x^2 = \frac{1}{3}$$

$$x = \pm \frac{1}{\sqrt{3}} \qquad \text{Racionalizando...}$$

$$x = \pm \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$x = \pm \frac{\sqrt{3}}{3}$$

04. Da lei de formação $a_{ij} = 2i - j$, temos que $a_{11} = 2 \cdot 1 - 1 = 1$ e que $a_{12} = 2 \cdot 1 - 2 = 0$, e assim por diante, temos que $A = \begin{pmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \\ 5 & 4 & 3 \end{pmatrix}$ e que $\det A = 0$.

05.

$$\begin{vmatrix} a & 1 & 0 \\ b & 3 & 0 \\ c & 4 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} a & 1 & 0 \\ b & 3 & 0 \\ c & 4 & 1 \end{vmatrix} \begin{matrix} a & 1 \\ b & 3 \\ c & 4 \end{matrix} = 0$$

$$3a - b = 0$$

$$b = 3a$$

06.
Temos que

$$\det A = \det B$$

$$\begin{vmatrix} x & 3 \\ y & 4 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 2 \\ y & 1 & 1 \\ x & 0 & 5 \end{vmatrix}$$

$$4x - 3y = 5 - x + 0 - 2x - 0 + 5y$$

$$7x - 8y = 5$$

Do sistema $\begin{cases} 7x - 8y = 5 \\ x + y = 5 \end{cases}$, temos que $x = 3$ e $y = 2$, logo $x^y = 3^2 = 9$.

07. Tomando a 1° coluna, por exemplo, pelo teorema de Laplace, o determinante será

$$a_{11} \cdot cof\,(a_{11}) + a_{21} \cdot cof\,(a_{21}) + a_{31} \cdot cof\,(a_{31}) + a_{41} \cdot cof\,(a_{41})$$

$$4 \cdot cof\,(a_{11}) + 0 \cdot cof\,(a_{21}) + 0 \cdot cof\,(a_{31}) + 0 \cdot cof\,(a_{41})$$

$$4 \cdot cof\,(a_{11}) + 0 + 0 + 0$$

$$4 \cdot cof\,(a_{11})$$

$$4 \cdot (-1)^{1+1} \cdot \begin{vmatrix} 1 & 0 & -5 \\ -4 & 0 & 4 \\ -3 & 1 & 1 \end{vmatrix}$$

$$4 \cdot (+1) \cdot (16)$$

$$64$$

Lembre-se que $cof\,(a_{ij}) = (-1)^{i+j} \cdot \Delta_{ij}$, onde Δ_{ij} é o determinante da matriz obtida eliminando a linha i e a coluna j.

08. Tomando a 2° coluna, por exemplo, pelo teorema de Laplace, o determinante será

$$a_{12} \cdot cof\,(a_{12}) + a_{22} \cdot cof\,(a_{22}) + a_{32} \cdot cof\,(a_{32}) + a_{42} \cdot cof\,(a_{42})$$

$$0 \cdot cof\,(a_{12}) + 0 \cdot cof\,(a_{22}) + 5 \cdot cof\,(a_{32}) + 0 \cdot cof\,(a_{42})$$

$$0 + 0 + 5 \cdot cof\,(a_{32}) + 0$$

$$5 \cdot cof\,(a_{32})$$

$$5 \cdot (-1)^{3+2} \cdot \begin{vmatrix} 1 & 3 & 2 \\ 3 & -1 & 0 \\ -1 & 3 & 0 \end{vmatrix}$$

$$5 \cdot (-1) \cdot (16)$$

$$-80$$

09. Tomando a 2° coluna, por exemplo, pelo teorema de Laplace, o determinante será

$$a_{12} \cdot cof\left(a_{12}\right) + a_{22} \cdot cof\left(a_{22}\right) + a_{32} \cdot cof\left(a_{32}\right) + a_{42} \cdot cof\left(a_{42}\right)$$
$$y \cdot cof\left(a_{12}\right) + 0 \cdot cof\left(a_{22}\right) + 0 \cdot cof\left(a_{32}\right) + y \cdot cof\left(a_{42}\right)$$
$$y \cdot cof\left(a_{12}\right) + 0 + 0 + y \cdot cof\left(a_{42}\right)$$
$$y \cdot cof\left(a_{12}\right) + y \cdot cof\left(a_{42}\right)$$
$$y \cdot (-1)^{1+2} \cdot \begin{vmatrix} x & 0 & w \\ x & z & 0 \\ x & 0 & 0 \end{vmatrix} + y \cdot (-1)^{4+2} \cdot \begin{vmatrix} 0 & z & w \\ x & 0 & w \\ x & z & 0 \end{vmatrix}$$
$$y \cdot (-1) \cdot (-xzw) + y \cdot (+1) \cdot (2xzw)$$
$$xyzw + 2xyzw$$
$$3xyzw$$

10. Tomando a 2° coluna, por exemplo, pelo teorema de Laplace, o determinante será

$$a_{12} \cdot cof\left(a_{12}\right) + a_{22} \cdot cof\left(a_{22}\right) + a_{32} \cdot cof\left(a_{32}\right) + a_{42} \cdot cof\left(a_{42}\right)$$
$$1 \cdot cof\left(a_{12}\right) + b \cdot cof\left(a_{22}\right) + 0 \cdot cof\left(a_{32}\right) + 1 \cdot cof\left(a_{42}\right)$$
$$1 \cdot cof\left(a_{12}\right) + b \cdot cof\left(a_{22}\right) + 0 + 1 \cdot cof\left(a_{42}\right)$$
$$1 \cdot cof\left(a_{12}\right) + b \cdot cof\left(a_{22}\right) + 1 \cdot cof\left(a_{42}\right)$$
$$1 \cdot (-1)^{1+2} \cdot \begin{vmatrix} a & c & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} + b \cdot (-1)^{2+2} \cdot \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} + 1 \cdot (-1)^{4+2} \cdot \begin{vmatrix} 1 & 0 & 1 \\ a & c & 0 \\ 1 & 1 & 1 \end{vmatrix}$$
$$1 \cdot (-1) \cdot (-c) + b \cdot (+1) \cdot (1) + 1 \cdot (+1) \cdot (a)$$
$$c + b + a$$
$$a + b + c$$

Gabarito

01. $x - y$
02. -1 ou 1/2
03. $\pm \frac{\sqrt{3}}{3}$
04. 0
05. $b = 3a$
06. 9
07. 64
08. -80
09. 3xyzw
10. $a + b + c$

Resolução comentada Lista 04

Capítulo 11 – Páginas 05 e 06
03.

$$\det M + \det(2M) + \det(3M) = \det M + 2^3 \cdot \det(M) + 3^3 \cdot \det(M) = \det M + 8 \cdot \det(M) + 27 \cdot \det(M) = 36 \cdot \det(M) = 36 \cdot 2 = 72$$

Gabarito E.

01. $\begin{vmatrix} 1 & 2 & 3 \\ a & 2a & 3a \\ b+1 & b+2 & b+3 \end{vmatrix} = 0$ para quaisquer valores de a e b, pois a 2° linha é proporcional a 1° Linha.

Gabarito A.

14. Resolvida na videoaula 03.
15. Resolvida na videoaula 03.
Exercícios extras

01.

$$\begin{vmatrix} 2 & 1 & -1 & 3 \\ 1 & 4 & -2 & 0 \\ 3 & -2 & 0 & 1 \\ -1 & 0 & 2 & 1 \end{vmatrix} = (-1)\cdot\begin{vmatrix} 1 & 4 & -2 & 0 \\ 2 & 1 & -1 & 3 \\ 3 & -2 & 0 & 1 \\ -1 & 0 & 2 & 1 \end{vmatrix} \overset{\text{(Chió)}}{=} (-1)\cdot\begin{vmatrix} 1-2\cdot 4 & -1-2\cdot(-2) & 3-2\cdot 0 \\ -2-3\cdot 4 & 0-3\cdot(-2) & 1-3\cdot 0 \\ 0-(-1)\cdot 4 & 2-(-1)\cdot(-2) & 1-(-1)\cdot 0 \end{vmatrix} = (-1)\cdot\begin{vmatrix} -7 & 3 & 3 \\ -14 & 6 & 1 \\ 4 & 0 & 1 \end{vmatrix} = (-1)\cdot(-60) = 60$$

02.

$$\begin{vmatrix} a & b & c \\ -2g & -2h & -2i \\ d & e & f \end{vmatrix} = (-2)\cdot\begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix} = (-2)\cdot(-1)\cdot\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (-2)\cdot(-1)\cdot 10 = 20$$

03. Note que como A é inversível, então $\det(A) \neq 0$.
Devemos ter

$$\det(3A) = \det(A^2)$$
$$3^3 \cdot \det A = (\det A)^2$$
$$(\det A)^2 - 27\cdot\det A = 0 \qquad \text{det A em evidência}$$
$$\det A\cdot(\det A - 27) = 0$$
$$\det A = 0 \text{ ou } \det A - 27 = 0$$
$$\det A = 0 \text{ ou } \qquad \det A = 27$$

Como $\det(A) \neq 0$, resta que $\det A = 27$.

04. $\det(A\cdot 2B) = \det(A)\cdot\det(2B) = \det(A)\cdot 2^3\cdot\det(B) = 3\cdot 2^3\cdot 4 = 96$

05.

$$\det(2A) = 2x - 150$$
$$2^4\cdot\det A = 2x - 150$$
$$16\cdot(-8) = 2x - 150$$
$$-128 = 2x - 150$$
$$x = 11$$

06. Seja n a ordem da matriz P. Temos que

$$\det(2P) = 96$$
$$2^n\cdot\det P = 96$$
$$2^n\cdot 3 = 96$$
$$2^n = \frac{96}{3}$$
$$2^n = 32$$
$$2^n = 2^5$$
$$n = 5$$